Semi-Quantum Cellular Automata With GPU Acceleration

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Objectives

What can a parallel classical computer hope to achieve from simulating quantum calculations:

- Provide an intuitive way to comprehend quantum computers.
- Study emergent quantum effects from quantum agent based models.
- Explore and aid in researching possible applications prior to the realisation of quantum computers.
- Increase performance compared to existing sequential algorithms by exploiting GPU architecture.

Introduction

Quantum Computing **(QC)** continues to be a thriving area of research between the disciplines of Physics and Computer Science, in the hope of creating the first quantum computer. Unlike classical bits (0 or 1), quantum bits (qubits) operate under the principle of superposition and therefore can exist in a probability of both states as shown in equations 1 and 2. This enables them to store an exponential amount of information due to quantum parallelism.

$$\Psi = \alpha_0|0\rangle + \alpha_1|1\rangle \quad (1)$$

Superposition state of a qubit.

$$|\alpha_0|^2 + |\alpha_1|^2 = 1 \quad (2)$$

Probability of measuring $|0\rangle or |1\rangle$.

However, not all algorithms will be suitable for QC and even those that meet the criteria will be drastically slower when simulated, compared to their classical counterpart. To emulate quantum operations, a likewise parallel system should be employed to fulfil the increased computational demand. Due to the inherently parallel nature of GPUs, it is a perfect candidate for accelerating these operations.

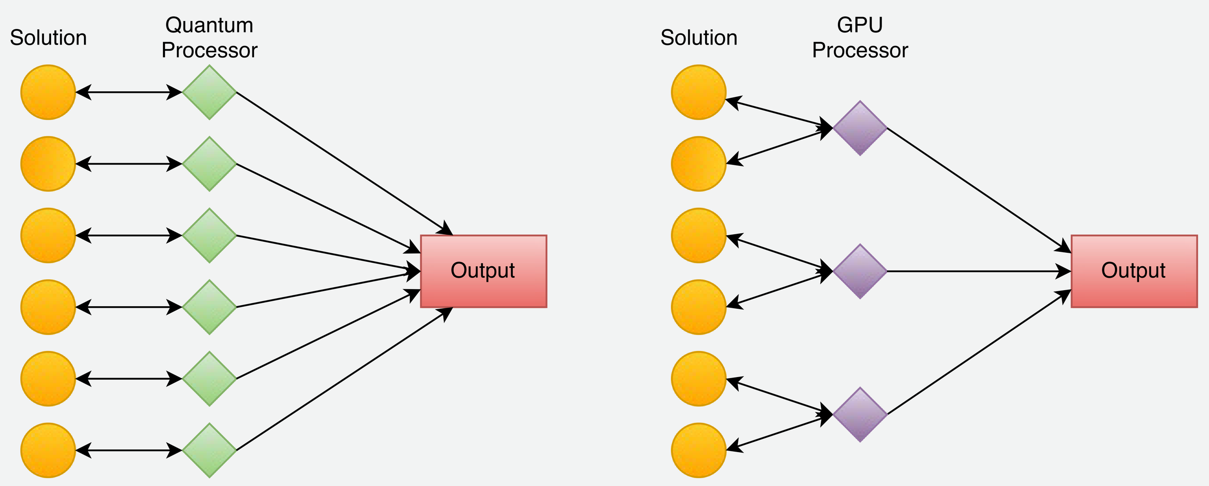

Figure 1: Quantum vs GPU

Quantum Cellular Automata

In 1985, David Deutsch released a paper detailing how a "universal quantum computer" might operate (Deutsch, 1985), similar to that proposed by Alan Turing for the classical computer (Turing, 1937). This was only the beginning of the enquiry into how quantum properties (superposition and interference) could be utilised for computational needs.

Almost from the inception, interest was shown in exploiting quantum behaviour within the bounds of a cellular automata (CA) (Grössing and Zeilinge, 1988). Much like it's classical counterpart, the aim is to study emergent behaviour and to better understand the real life consequences of such actions in a confined and observable manner.

Methods

Simulating a QC system is both computationally and memory intensive due to it's exponential nature (Niwa, Matsumoto, and Imai, 2002). As such, special attention must be made to ensure maximum parallelisation and efficient memory accesses.

- Utilise CUDA to exploit GPU hardware for massively parallel processing.
- Algebraic manipulations - reduce floating point operations by combining suitable calculations.
- Coalesced and low latency memory (shared) for life/qubit data.

Definition

"Thus we will reserve the adjective 'quantum' for CA with exactly unitary, nontrivial, local evolution" (D. Meyer, 1996).

Semi-Quantum Game of Life

The original Game of Life (GoL) (Gardner, 1970) became one of the most widely studied CA due to its simple rules, balanced states and interesting emergent patterns.

In contrast to the classical CA, QGoL utilises qubits instead of binary states, with oscillators corresponding to the phase. This oscillator can be stored as a complex number $Ae^{(i\varphi)}$, A and φ being real positive numbers. Normalisation must take place to ensure the probability equation 2 is correct.

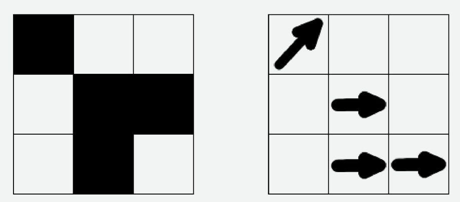

Figure 2: Classical vs Quantum

Since the cells can exist in superposition, the regular rules for GoL change slightly to incorporate a non-integer number of alive neighbours. The updated QGoL rules are shown in Table 1, $D = Death, B = Birth, S = Unchanged$.

Neighbours	Operator
$N \leqslant 1$	D
$1 < N \leqslant 2$	$(\sqrt{2}+1)(2-N)D + (N-1)S$
$2 < N \leqslant 3$	$(\sqrt{2}+1)(3-N)S + (N-2)B$
$3 < N < 4$	$(\sqrt{2}+1)(4-N)B + (N-3)D$
$N \geqslant 4$	D

Table 1: QGoL Rules

Results

Alive neighbours are calculated by taking the modulus of the sum of surrounding states, see equation 3.

$$\sum_{j=1}^{8} |1\rangle_j = Ae^{i\varphi} \quad (3)$$

Neighbour count.

A suitable combination of operators are calculated from table 1, the individual components of which are listed in equations 4 and 5.

$$B\begin{pmatrix}|0\rangle \\ |1\rangle\end{pmatrix} = \begin{pmatrix}0 \\ |1\rangle + ||0\rangle|e^{i\varphi}\end{pmatrix} \quad (4)$$

Birth Operator.

$$D\begin{pmatrix}|0\rangle \\ |1\rangle\end{pmatrix} = \begin{pmatrix}|0\rangle + ||1\rangle|e^{i\varphi} \\ 0\end{pmatrix} \quad (5)$$

Death Operator.

An example of a single iteration within QGoL is shown in figure 3. This demonstrates the combination of surrounding phases, resulting in a stable pattern.

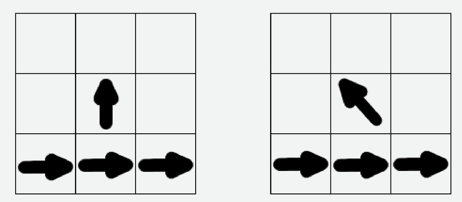

Figure 3: A single QGoL iteration.

Conclusion and Future Directions

Quantum simulations are a vital part in the advancement of the field by providing an intuitive method of observing phenomenon which are usually unobservable. Agent based models, and game theory by extension, has proved applicable in various disciplines from pharmacology (Cosgrove et al., 2015) to economics (Nature, 2009). Due to the potential in extending these systems to include quantum behaviour, further research will be carried out on this subject. The aim is to implement various simulations, at higher dimensions, with an increased grid size to study the emergent properties on a larger scale.

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